Doppler Spread Estimation by Tracking the Delay-Subspace for OFDM Systems in Doubly Selective Fading Channels

Xiaochuan Zhao, *Student Member, IEEE,* Tao Peng, *Member, IEEE,* Ming Yang and Wenbo Wang, *Member, IEEE*

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Abstract—In this paper, a novel maximum Doppler spread estimation algorithm is presented for OFDM systems with the comb-type pilot pattern in doubly selective fading channels. First, the least-squared estimated channel frequency responses on pilot tones are used to generate two auto-correlation matrices with different lags. Then, according to these two matrices, a Doppler dependent parameter is measured. Based on a time-varying multipath channel model, the parameter is expanded and then transformed into a non-linear high-order polynomial equation, from which the maximum Doppler spread is readily solved by using the Newton's method. The delay-subspace is utilized to reduce the noise that biases the estimator. Besides, the subspace tracking algorithm is adopted as well to automatically update the delay-subspace. Simulation results demonstrate that the proposed algorithm converges for a wide range of SNR's and Doppler's.

Index Terms—Doppler spread, Estimation, Subspace tracking, OFDM, Doubly selective fading channels, Comb-type pilot.

I. INTRODUCTION

The maximum Doppler spread is one of the key parameters for the adaptive strategies to tune mobile communication systems to accommodate various radio transmission environments [1] and, especially, alleviate the inter-carrier interference (ICI) for orthogonal frequency division multiplexing (OFDM) systems. Most methods of estimating the maximum Doppler spread reported in literatures are categorized into two classes [2]: level-crossing-rate-based and covariance-based techniques. Since the algorithms reviewed in [2], such as [3], were not specifically designed for OFDM systems, they did not exploit the special signal structure of OFDM systems. On the other hand, most algorithms designed for OFDM systems are covariance-based. [4] determined the maximum Doppler spread through estimating the smallest positive zero crossing point. Cai [5] proposed to obtain the time correlation function (TCF) by exploiting the cyclic prefix (CP) and its counterpart. However, Yucek [6] commented that for scalable OFDM systems whose CP sizes were varying along the time, it was difficult for [5] to sufficiently estimate the TCF, therefore

its accuracy would be degraded significantly. In stead, Yucek proposed to estimate the maximum Doppler spread by taking advantage of the periodic training symbols. However, in order to reduce overheads, training symbols are sparse and typically transmitted as preambles to facilitate the frame timing and carrier frequency synchronization, which would cause [6] to converge slowly or even fail.

In this paper, we propose to estimate the maximum Doppler spread by exploiting the comb-type pilot tones [7] that are widely adopted by wireless standards. The channel frequency responses (CFR's) estimated from the pilot tones are projected onto the delay-subspace [8] to reduce the noise perturbation and then used to acquire a Doppler dependent parameter. With a careful expansion of the parameter, a nonlinear high-order polynomial equation is formed, from which the maximum Doppler spread is readily solved by resorting to the Newton's iteration. Moreover, the subspace tracking algorithm [9] is adopted as well to track the drifting delay-subspace.

This paper is organized as follows. In Section II, the OFDM system and channel model are introduced. Then, the maximum Doppler spread estimation algorithm is presented in Section III. Simulation results and analyses are provided in Section IV. Finally, Section V concludes the paper.

A. Basic Notation

Uppercase and lowercase boldface letters denote matrices and column vectors, respectively. $(\cdot)^H$ and $||\cdot||_F$ denote conjugate transposition and Frobenius norm, respectively. $E(\cdot)$ represents the mathematical expectation of a stochastic process. $[\cdot]_i$ and $[\cdot]_{i,j}$ denote the i-th and (i,j)-th elements of a vector and a matrix, respectively. $diag(\mathbf{A})$ denotes a diagonal matrix with the diagonal elements of $\mathbf{A}$ on the diagonal.

II. SYSTEM MODEL

Consider an OFDM system with a bandwidth of $BW = 1/T$ Hz (T is the sampling period). N denotes the total number of tones, and a CP of length L_{cp} is inserted before each symbol to eliminate the inter-block interference. Thus, the whole symbol duration is $T_s = (1+r_{cp})NT$, where $r_{cp} = \frac{L_{cp}}{N}$. In each OFDM symbol, P tones are transmitted as pilots to assist channel estimation. In addition, the optimal pilot pattern, i.e., the equipowered and equispaced [10], is assumed.

The complex baseband model of a linear time-variant mobile channel with L paths can be described by [11]

$$h(t,\tau) = \sum\nolimits_{l=0}^{L-1} h_l(t)\delta(\tau - \tau_l T) \qquad (1)$$

where $\tau_l \in \mathcal{R}$ is the non-sample-spaced delay of the l-th path normalized to the sampling period, and $h_l(t)$ is the corresponding complex amplitude. According to the assumption of the wide-sense stationary uncorrelated scattering, $h_l(t)$'s are modeled as uncorrelated narrowband complex Gaussian processes. In the sequel, $P \geq L$ is assumed for determinability. Furthermore, by assuming the uniform scattering environment introduced by Clarke [12], all $h_l(t)$'s have the identical normalized TCF, therefore the TCF of the l-th path is

$$r_{t,l}(\Delta t) = \sigma_l^2 J_0\left(2\pi f_d \Delta t\right) \tag{2}$$

where σ_l^2 is the power of the l-th path, f_d is the maximum Doppler spread, and $J_0(\cdot)$ is the zeroth order Bessel function of the first kind. Moreover, the total power of the channel is normalized, i.e., $\sum_{l=0}^{L-1} \sigma_l^2 = 1$.

Assuming a sufficient CP, i.e., $L_{cp} \geq L$, the discrete signal model in the frequency domain is

$$\mathbf{y}_f(n) = \mathbf{H}_f(n)\mathbf{x}_f(n) + \mathbf{n}_f(n) \tag{3}$$

where $\mathbf{x}_f(n), \mathbf{y}_f(n), \mathbf{n}_f(n) \in \mathcal{C}^{N \times 1}$ are the n-th transmitted and received signal and additive white Gaussian noise (AWGN) vectors, respectively, and $\mathbf{H}_f(n) \in \mathcal{C}^{N \times N}$ is the channel transfer matrix whose $(k+\nu, k)$-th element is

$$[\mathbf{H}_f(n)]_{k+\upsilon,k} = \frac{1}{N} \sum_{m=0}^{N-1} \sum_{l=0}^{L-1} h_l(n,m) e^{-j2\pi(\upsilon m + k\tau_l)/N}$$

where $h_l(n,m) = h_l(nT_s + (L_{cp} + m)T)$ is the sampled complex amplitude of the l-th path, and k and υ denote frequency and Doppler spread, respectively. Apparently, ICI is present due to the non-diagonal $\mathbf{H}_f(n)$. However, when $f_d T_s \leq 0.1$, the signal-to-interference ratio (SIR) is over 17.8 dB [13], which enables us to discard non-diagonal elements of $\mathbf{H}_f(n)$ with a negligible performance penalty.

For the comb-type pilot pattern, only pilot tones, denoted as $\mathbf{y}_p(n) \in \mathcal{C}^{P \times 1}$, are extracted from $\mathbf{y}_f(n)$. Then, approximating $\mathbf{H}_f(n)$ by a diagonal matrix, (3) is modified to

$$\mathbf{y}_p(n) = \mathbf{X}_p(n)\mathbf{h}_p(n) + \mathbf{n}_p(n) \tag{4}$$

where $\mathbf{X}_p(n) \in \mathcal{C}^{P \times P}$ is a pre-known diagonal matrix, and $\mathbf{h}_p(n) \in \mathcal{C}^{P \times 1}$ consists of the diagonal elements of $\mathbf{H}_f(n)$. By denoting the instantaneous CFR as $H(n,m,k) = \sum_{l=0}^{L-1} h_l(n,m) e^{-j2\pi k\tau_l/N}$, we have $[\mathbf{h}_p(n)]_p = \frac{1}{N} \sum_{m=0}^{N-1} H(n,m,\theta_p)$, where θ_p is the index of the p-th pilot tone. Hence, $\mathbf{h}_p(n)$ is the time-averaging CFR during the n-th OFDM symbol. Besides, the noise term $\mathbf{n}_p(n) \sim \mathcal{CN}(0, \sigma_n^2 \mathbf{I}_P)$.

III. Maximum Doppler Spread Estimation

First of all, the least-squared (LS) channel estimation on pilot tones is carried out at the receiver, that is,

$$\mathbf{h}_{p,ls}(n) = \mathbf{X}_p(n)^{-1}\mathbf{y}_p(n) = \mathbf{h}_p(n) + \mathbf{w}_p(n) \tag{5}$$

where $\mathbf{h}_{p,ls}(n) \in \mathcal{C}^{P \times 1}$ is the LS-estimated time-averaging CFR, and $\mathbf{w}_p(n) = \mathbf{X}_p(n)^{-1}\mathbf{n}_p(n)$ is the noise term. For equipowered and PSK-modulated pilot tones, $\mathbf{X}_p(n)^H \mathbf{X}_p(n) = \mathbf{I}_P$, therefore $\mathbf{w}_p(n) \sim \mathcal{CN}(0, \sigma_n^2 \mathbf{I}_P)$.

By defining the Fourier transform matrix $\mathbf{F}_{p,\tau} \in \mathcal{C}^{P \times L}$ as $[\mathbf{F}_{p,\tau}]_{p,l} = e^{-j2\pi\theta_p\tau_l/N}$, the 0-lag auto-correlation matrix is

$$\mathbf{R}_h(0) \triangleq E\left[\mathbf{h}_{p,ls}(n)\mathbf{h}_{p,ls}(n)^H\right] = \xi_0 \mathbf{F}_{p,\tau}\mathbf{D}\mathbf{F}_{p,\tau}^H + \sigma_n^2 \mathbf{I}_P \tag{6}$$

where $\mathbf{D} \triangleq diag(\sigma_l^2)$, $l = 0, \ldots, L-1$, and ξ_0 is

$$\xi_0 \triangleq \frac{1}{N^2} \sum_{m=0}^{N-1} \sum_{q=0}^{N-1} J_0(2\pi f_d(m-q)T) \tag{7}$$

Similarly, the β-lag auto-correlation matrix of $\mathbf{h}_{p,ls}(n)$ is

$$\mathbf{R}_h(\beta) \triangleq E\left[\mathbf{h}_{p,ls}(n+\beta)\mathbf{h}_{p,ls}(n)^H\right] = \xi_\beta \mathbf{F}_{p,\tau}\mathbf{D}\mathbf{F}_{p,\tau}^H \tag{8}$$

where ξ_β is

$$\xi_\beta \triangleq \frac{1}{N^2} \sum_{m=0}^{N-1} \sum_{q=0}^{N-1} J_0(2\pi f_d(m-q+\beta(1+r_{cp})N)T) \tag{9}$$

If the number of channel taps is known, the delay-subspace, denoted as $\mathbf{U}_{\tau,s} \in \mathcal{C}^{P \times L}$, and the variance of noise σ_n^2 can be acquired with the eigenvalue decomposition. If unknown, however, the number of significant taps can be estimated by applying the minimum description length (MDL) algorithm [14]. Then, a Doppler dependent parameter is defined as

$$\eta \triangleq \sqrt{\frac{||\mathbf{U}_{\tau,s}^H \mathbf{R}_h(\beta)\mathbf{U}_{\tau,s}||_F^2}{||\mathbf{U}_{\tau,s}^H \mathbf{R}_h(0)\mathbf{U}_{\tau,s} - \sigma_n^2 \mathbf{I}_L||_F^2}} = \frac{\xi_\beta}{\xi_0} \tag{10}$$

Through some manipulations (see Appendix A), ξ_0 and ξ_β can be expanded into series and approximately expressed as

$$\xi_0 = \sum_{k=0}^{\infty} s_k \approx \sum_{k=0}^{\infty} \frac{(-\psi^2)^k}{k!(k+1)!(2k+1)} \tag{11}$$

$$\xi_\beta = \sum_{k=0}^{\infty} t_k \approx \sum_{k=0}^{\infty} \frac{(-\psi^2)^k}{k!(k+1)!(2k+1)} \times \frac{1}{2}\left[(1+\varphi)^{2k+2} + (1-\varphi)^{2k+2} - 2\varphi^{2k+2}\right] \tag{12}$$

where $N \gg 1$, $\psi = \pi f_d NT$, and $\varphi = \beta(1+r_{cp})$.

Since the absolute values of s_k and t_k decrease very fast, a finite number, say, K, of terms in (11) and (12) are sufficient to meet the accuracy requirement. So, with (10)–(12), we have

$$\sum_{k=0}^{K-1} (t_k - \eta s_k) = 0 \tag{13}$$

Let $x = -\psi^2$ and $c_k = \frac{[(1+\varphi)^{2k+2}+(1-\varphi)^{2k+2}-2\varphi^{2k+2}]-2\eta}{2k!(k+1)!(2k+1)}$. Then, (13) is equivalent to

$$\sum_{k=0}^{K-1} c_k x^k = 0 \tag{14}$$

(14) is a non-linear high-order polynomial equation. By resorting to Newton's method, its root, denoted as x^*, is readily solved after numbers of iterations. Accordingly, f_d is given by

$$f_d = \frac{\sqrt{-x^*}}{\pi NT} \tag{15}$$

When the transceiver is moving, the path delays of the channel are slowly drifting [15] [8], which causes $\mathbf{F}_{p;\tau}$ to vary and so does $\mathbf{U}_{\tau,s}$. To accommodate the variation, the subspace tracking algorithm is adopted to automatically update $\mathbf{U}_{\tau,s}$.

The proposed algorithm is summarized in the following. It is worth noting that its computation complexity depends on the QR-decomposition operation that is $\mathcal{O}(P \times L^2)$. For sparse multipath channels, the complexity is moderate because L is quite small. Besides, the choice of K in (14) can be made according to the tradeoff between the accuracy and complexity. After K is chosen, the convergence of Newton's method is quadratic along the number of iterations. In fact, numerical results show that less than 4 iterations are sufficient to achieve a precision of 10^{-4}. Moreover, for each iteration, the first differential is readily obtained thanks to the polynomial coefficients, which reduces the complexity of Newton's method considerably. Finally, as ξ_β is oscillatingly attenuating along β, (14) has multiple roots for large β [1]. In order to converge to the specific root, the initialization of Newton's iteration, therefore, should be carefully chosen according to β.

Initialize: $(n = 0)$
$\mathbf{Q}_0(0) = \mathbf{Q}_\beta(0) = [\mathbf{I}_{L_m}, \mathbf{0}_{L_m, P-L_m}^T]^T$
$\mathbf{A}_0(0) = \mathbf{A}_\beta(0) = \mathbf{0}_{P, L_m}, \quad \mathbf{C}_0(0) = \mathbf{C}_\beta(0) = \mathbf{I}_{L_m}$
Run: $(n = n + 1)$
Input: $\mathbf{h}_{p,ls}(n)$
Step 1: Updating eigenvalues of $\mathbf{R}_h(0)$:
$\mathbf{Z}_0(n) = \mathbf{h}_{p,ls}(n)\mathbf{h}_{p,ls}(n)^H$
$\mathbf{A}_0(n) = \alpha\mathbf{A}_0(n-1)\mathbf{C}_0(n-1) + (1-\alpha)\mathbf{Z}_0(n)\mathbf{Q}_0(n-1)^H$
$\mathbf{A}_0(n) = \mathbf{Q}_0(n)\mathbf{R}_0(n) : \textbf{\textit{QR-factorization}}$
$\mathbf{C}_0(n) = \mathbf{Q}_0(n-1)^H\mathbf{Q}_0(n)$

Step 2: Updating eigenvalues of $\mathbf{R}_h(\beta)$:
$\mathbf{Z}_\beta(n) = \mathbf{h}_{p,ls}(n)\mathbf{h}_{p,ls}(n-\beta)^H$
$\mathbf{A}_\beta(n) = \alpha\mathbf{A}_\beta(n-1)\mathbf{C}_\beta(n-1) + (1-\alpha)\mathbf{Z}_\beta(n)\mathbf{Q}_\beta(n-1)^H$
$\mathbf{A}_\beta(n) = \mathbf{Q}_\beta(n)\mathbf{R}_\beta(n) : \textbf{\textit{QR-factorization}}$
$\mathbf{C}_\beta(n) = \mathbf{Q}_\beta(n-1)^H\mathbf{Q}_\beta(n)$

Step 3: Estimating L and σ_n^2:
$\hat{L} = MDL(diag(\mathbf{R}_0(n))), \hat{\sigma}_n^2 = \frac{1}{P-\hat{L}}\sum_{p=\hat{L}+1}^{P}[\mathbf{R}_0(n)]_{p,p}$
Step 4: Estimating η according to (10):
$$\hat{\eta} = \sqrt{\frac{\sum_{l=1}^{\hat{L}(n)}\left|[\mathbf{R}_\beta(n)]_{l,l}\right|^2}{\sum_{l=1}^{\hat{L}(n)}\left|[\mathbf{R}_0(n)]_{l,l} - \hat{\sigma}_n^2\right|^2}}$$
Step 5: Estimating f_d by (14) and (15).
Remark: α is an exponential forgetting factor close to 1. L_m is the maximum rank to be tested. $MDL(\cdot)$ denotes the MDL detector. In simulations, we set $\alpha = 0.995$, $\beta = 1$, $L_m = 10$, $K = 8$, and the precision threshold of Newton iteration and the maximum number of iterations as 10^{-4} and 4, respectively.

IV. SIMULATION RESULTS

The performance of the proposed algorithm is evaluated for an OFDM system with $BW = 12$ MHz ($T = 1/BW = 83.3$ ns), $N = 1024$, $L_{cp} = 128$ and $P = 128$. Two 3GPP E-UTRA channel models are adopted: Extended Vehicular A model (EVA) and Extended Typical Urban model (ETU) [16]. The delay profile of EVA is [0, 30, 150, 310, 370, 710, 1090,

1730, 2510] ns, and its power profile is [0.0, −1.5, −1.4, −3.6, −0.6, −9.1, −7.0, −12.0, −16.9] dB. For ETU, they are [0, 50, 120, 200, 230, 500, 1600, 2300, 5000] ns and [−1.0, −1.0, −1.0, 0.0, 0.0, 0.0, −3.0, −5.0, −7.0] dB, respectively. The classic Doppler spectrum, i.e., Jakes' spectrum [11], is applied to generate the Rayleigh fading channels.

In Fig.1, the proposed algorithm is evaluated for $f_d = 200$, 400, and 600 Hz, respectively, within a wide range of SNR's. And two durations of observation (20 and 40 ms) are adopted to acquire the sample auto-correlation matrices. From the figure, it is evident that the performance of the proposed algorithm is robust when $SNR \geq 5$ dB, since the delay-subspace effectively reduces the noise disturbance. Furthermore, the estimation accuracy is rather high for $f_d \geq 400$ Hz and the 40 ms observation but somewhat deteriorated for 200 Hz and 20 ms. It is accounted for the accumulation process of the sample matrices: the larger the maximum Doppler spread is, the faster the CFR updates; the longer the duration is, the more sufficient the sample matrices are. When f_d is yet smaller, say, 50 Hz, the proposed algorithm, nevertheless, may fail with the 40 ms observation due to the insufficient sample matrices. Besides, a simplified version of the proposed algorithm [17] still outperforms the CP-based algorithm [5].

The convergence of the proposed algorithm is verified with various Doppler's, i.e., $f_d = 600$, 400, 200 Hz, when $SNR = 15$ dB for EVA and ETU channels, respectively. As plotted in Fig.2, the estimated f_d's converge after hundreds of samples. Furthermore, since the CFR updates faster when f_d is larger, the convergence speed is faster for larger f_d than smaller. In addition, for all the curves drawn in Fig.2, the estimated values fluctuate around their true ones within a certain range, and the variations are larger for smaller f_d's because of the sensitivity to the estimation error of η in (10) when $f_d T_s$ is small. If necessary, an averaging/smoothing window over the output of the proposed algorithm can be applied to supply a more stable estimation.

V. CONCLUSIONS

The maximum Doppler spread is crucial for adaptive strategies in OFDM systems. In this paper, we propose a subspace-based maximum Doppler spread estimation algorithm applicable to the comb-type pilot pattern. By tracking the drifting delay-subspace, the noise is greatly reduced. And by solving the polynomial equation with the Newton's method, high accuracy can be achieved with moderate complexity. The performance of the proposed algorithm is demonstrated to be robust over a wide range of SNR's and Doppler's by simulations. Besides, the proposed algorithm can be readily integrated into the channel estimators with the subspace tracker [8] [18], which lends a broad application promise to it.

APPENDIX A
APPROXIMATIONS OF ξ_0 AND ξ_β

The Maclakutin series of $J_0(z)$ is [19]

$$J_0(z) = \sum_{k=0}^{\infty}\frac{(-1)^k}{2^{2k}(k!)^2}z^{2k} \qquad (16)$$

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[1]For applied OFDM systems, (14) has only one negative root close to zero when $\beta \leq 4$.
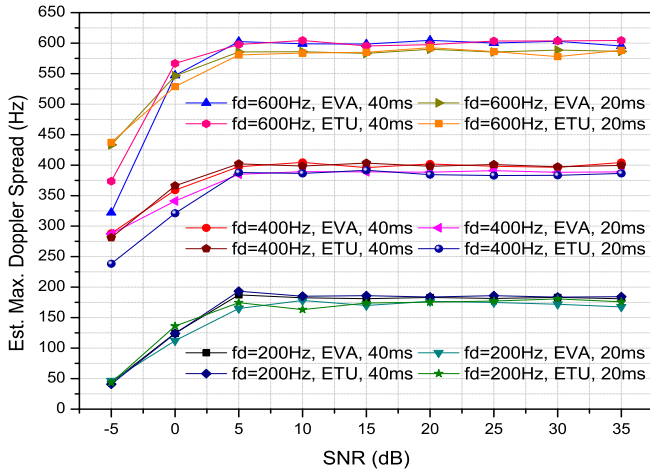
Fig. 1. The performance evaluation for the proposed algorithms for EVA and ETU channels with different amounts of samples and Doppler's.

Fig. 2. The convergence of the proposed algorithm when $SNR = 15$ dB and $\beta = 1$ for EVA and ETU channels.

hence (9) is rewritten as

$$\xi_\beta = \sum_{k=0}^{\infty} t_k = \sum_{k=0}^{\infty} \frac{(-1)^k}{(k!N)^2} \sum_{m_1=0}^{N-1} \sum_{m_2=0}^{N-1} \{\pi f_d T [(m_1 - m_2) + \beta(1 + r_{cp})N]\}^{2k} \quad (17)$$

Denoting $\psi = \pi f_d NT$ and $\varphi = \beta(1 + r_{cp})$, t_k is further expanded as

$$t_k = \frac{(-\psi^2)^k}{(k!N)^2} \sum_{m_1=0}^{N-1} \sum_{m_2=0}^{N-1} \sum_{p=0}^{2k} C_{2k}^p \varphi^{2k-p} \times$$
$$\sum_{q=0}^{p} C_p^q (\frac{m_1}{N})^q (-\frac{m_2}{N})^{p-q}$$
$$= \frac{(-\psi^2)^k}{(k!)^2} \sum_{p=0}^{2k} C_{2k}^p \varphi^{2k-p} \sum_{q=0}^{p} C_p^q \times$$
$$\left[\frac{1}{N} \sum_{m_1=0}^{N-1} (\frac{m_1}{N})^q\right] \left[\frac{1}{N} \sum_{m_2=0}^{N-1} (-\frac{m_2}{N})^{p-q}\right] \quad (18)$$

When $N \gg 1$, we have

$$\frac{1}{N} \sum_{m_1=0}^{N-1} (\frac{m_1}{N})^q \approx \frac{1}{q+1}, \quad \frac{1}{N} \sum_{m_2=0}^{N-1} (-\frac{m_2}{N})^{p-q} \approx \frac{(-1)^{p-q}}{p-q+1}$$

Thus, t_k can be approximated as

$$t_k \approx \frac{(-\psi^2)^k}{(k!)^2} \sum_{p=0}^{2k} C_{2k}^p \varphi^{2k-p} \frac{1 + (-1)^p}{(p+1)(p+2)}$$
$$= \frac{(-\psi^2)^k}{k!(k+1)!(2k+1)} \times$$
$$\frac{1}{2} \left[(1 + \varphi)^{2k+2} + (1 - \varphi)^{2k+2} - 2\varphi^{2k+2}\right] \quad (19)$$

Similarly, by letting $\beta = 0$ in (19), (7) is expanded and approximated as

$$\xi_0 = \sum_{k=0}^{\infty} s_k \approx \sum_{k=0}^{\infty} \frac{(-\psi^2)^k}{k!(k+1)!(2k+1)} \quad (20)$$
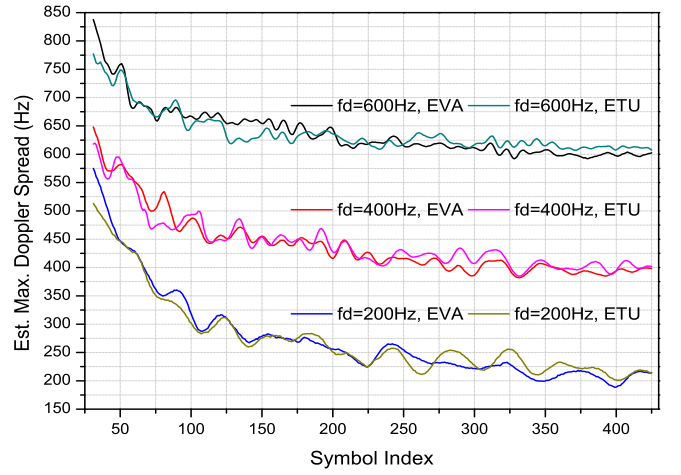
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